# Written Exam at the Department of Economics Summer 2018 

## Advanced Industrial Organization

Final Exam

May 29, 2018.
(3-hour closed book exam)

Answers only in English.

## This exam question consists of 4 pages in total (including the current page)

NB: If you fall ill during an examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. In this connection, you must complete a form. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

## Be careful not to cheat at exams!

- You cheat at an exam, if during the exam, you:
- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

For all the questions, please explain briefly how you obtain your answer and what is the logic behind it.

## Question 1

A profit-maximizing seller has $Q$ units of a perfectly divisible good for sale, where $0<$ $Q<6$. As he already has produced the $Q$ units of the good, all production costs are sunk at the time of pricing (and there are no costs of selling). There are two markets to which he can sell, i.e. he sells $q_{1} \geq 0$ to market 1 and $q_{2} \geq 0$ to market 2 where $q_{1}+q_{2} \leq Q$. The inverse demand functions in the two markets are as follows:

- market 1: $P_{1}\left(q_{1}\right)=1-q_{1}\left(\right.$ for $\left.q_{1} \in[0,1]\right)$
- market 2: $P_{2}\left(q_{2}\right)=5-q_{2}\left(\right.$ for $\left.q_{2} \in[0,5]\right)$
(a) Which quantity will the seller sell to each of the two markets? What will be the prices in the two markets? (hint: your answer will depend on the value of $Q$ )
(b) Which allocation of $Q$ over the two markets maximizes welfare (where welfare is defined as the sum of consumer surplus in the two markets and the seller's surplus)? Is the profit maximizing allocation from the previous subquestion welfare maximizing?
(c) Describe briefly how this model of a multimarket monopolist relates to independent private value auctions - both in terms of modelling and in terms of results. (hint: you are not expected to calculate something here. In particular, you do not have to derive the optimal auction but you can base your discussion on the results derived in the lecture.)


## Question 2

Consider a market with two firms selling horizontally differentiated products. Firm 1 and Firm 2 are located at different ends of the Hotelling line, where Firm 1 sets price $p_{1}$ for product 1 and Firm 2 sets price $p_{2}$ for product 2. There is a unit measure of consumers, each characterized by location parameter $x$, where $x$ is uniformly distributed on the interval $[0,1]$. Each consumer has unit demand.

Consumers are loss averse, and Firm 1 is most prominent in the market, so that all consumers take product 1 as their reference product. Specifically, given prices $p_{1}$ and $p_{2}$, consumer $x$ values product 1 at

$$
\begin{equation*}
v-x-p_{1}, \tag{1}
\end{equation*}
$$

and values product 2 at

$$
\begin{equation*}
v-(1-x)-p_{2}-\left(\lambda_{p}-1\right) \max \left(0, p_{2}-p_{1}\right)-\left(\lambda_{t}-1\right) \max (0,1-2 x), \tag{2}
\end{equation*}
$$

where $v>0, \lambda_{p} \geq 1$, and $\lambda_{t} \geq 1$.
Given prices $p_{1}$ and $p_{2}$, denote demand for product 1 by $q_{1}\left(p_{1}, p_{2}\right)$ and demand for product 2 by $q_{2}\left(p_{1}, p_{2}\right)$. Throughout this question, you can assume that the market is fully covered, so that $q_{1}\left(p_{1}, p_{2}\right)+q_{2}\left(p_{1}, p_{2}\right)=1$, and that both firms have a strictly positive market share, $0<q_{1}\left(p_{1}, p_{2}\right)<1$, and $0<q_{2}\left(p_{1}, p_{2}\right)<1$.
(a) Briefly explain how expression (2) can capture consumer loss aversion in both the 'price dimension' and in the 'product match dimension'.

For the rest of this question, assume that $\lambda_{p}>1$ and $\lambda_{t}=1$, so that consumers are only loss averse in one dimension. Thus, expression (2) reduces to

$$
\begin{equation*}
v-(1-x)-p_{2}-\left(\lambda_{p}-1\right) \max \left(0, p_{2}-p_{1}\right) . \tag{3}
\end{equation*}
$$

(b) Using expressions (1) and (3), show explicitly that demand for product 1 when $p_{1} \geq p_{2}$ is given by $q_{1}\left(p_{1}, p_{2}\right)=\frac{1}{2}\left(1+p_{2}-p_{1}\right)$.
(c) Using expressions (1) and (3), derive an expression for demand for product 1 when $p_{1}<p_{2}$.
(d) Using your answers from parts (b) and (c), argue whether demand is more or less price sensitive than in a setting without consumer loss aversion (i.e. in a setting where $\lambda_{p}=\lambda_{t}=1$ ). Briefly give intuition, in words, as to whether your argument would likely change if we had instead assumed $\lambda_{p}=1$ and $\lambda_{t}>1$ throughout the question. Please attempt to answer even if you did not successfully complete earlier parts of this question.

## Question 3

Consider a market with two competing technologies, $A$-tech and $B$-tech. The two technologies have the same features, but they are non-compatible.

There are two periods, $t=1,2$. In each period there are 50 consumers, who each wish to buy one unit of either the $A$-tech or the $B$-tech. 1st-period consumers can only buy in period 1 (or not at all) and 2nd-period consumers can only buy in period 2 (or not at all).

Consumer utility features network effects: if a consumer buys a particular tech $i \in$ $\{A, B\}$, her utility depends on the total number of consumers, $N_{i}$, who buy that tech. The world is simple, so she gets one util per consumer who buys that technology:

$$
v=N_{i} \cdot 1
$$

The utility does not depend on which period the other consumers buy, but only on how many buy in total over the two periods. If a consumer does not buy, she receives zero utility. Consumers in this exercise are smart, they have rational expectations.

The $A$-tech has a marginal cost $M C_{A 1}=50$ in period 1 and $M C_{A 2}=50$ in period 2. The $B$-tech has marginal cost $M C_{B 1}=60$ in period 1 and $M C_{B 2}=25$ in period 2 . We first consider the case where there are no patents, so both technologies will be sold at marginal cost in both periods (if sold at all).
(a) Suppose first-period consumers all choose $A$, and consider play in the second period. Find the set of second-period equilibria. If there are multiple equilibria, find the Pareto optimal one for second-period consumers.
(b) Suppose first-period consumers all choose $B$, and consider play in the second period. Show that in this case, all second-period consumers will also choose $B$ in equilibrium.
In the rest of the exercise, we suppose that consumers in the second period coordinate on the second-period equilibrium which is optimal for them, given the choice of firstperiod consumers.
(c) Consider play in the first period. To simplify our life, we assume that first-period consumers coordinate on the equilibrium which is optimal for them. Find this firstperiod equilibrium, and combining with the answers from (a) and (b), state the overall equilibrium (i.e considering both period-1 and period-2 consumers).
(d) Now look at second-best pricing. That is, imagine a planner wishes to maximize total utility and can set prices (for instance through taxes and subsidies) but she cannot force consumers to make specific choices. We will refer to the resulting outcome after the planner has chosen prices to maximize total utility as the socially optimal technology choices.
Find the socially optimal technology choices in the two periods. Are they the same as the overall equilibrium outcome found in (c)? If not, explain the intuition for what happens.
(e) Now suppose the $B$-tech is patented, so that the firm producing the $B$-tech can set a price in each period which is potentially different from marginal cost (above or below as it wishes). The $A$-tech not patented and sold at marginal cost (if sold at all).
What are the profit maximizing prices for $B$ in the two periods? Which products are sold in the two periods?

